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A NEW FUZZY TIME SERIES ANALYSIS APPROACH BY USING DIFFERENTIAL EVOLUTION ALGORITHM AND CHRONOLOGICALLY-DETERMINED WEIGHTS[*]

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Abstract

Fuzzy time series approaches, which do not require the strict assumptions of traditional time series approaches, generally consist of three stages. These stages are called as the fuzzification of crisp time series observations, the identification of fuzzy relationships and the defuzzification, respectively. All of these stages play an important role on the forecasting performance of the model. By this study we want to contribute to the stage of fuzzification so that the interval length is determined by using the differential evolution algorithm and also we take into account chronological-determined weights in the stage of defuzzification.

Keywords: Fuzzy time series, Fuzzification, Differential evolution algorithm, Forecasting

JEL Classification: C53 - Forecasting and Prediction Methods

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1. Introduction

The fuzzy set was firstly introduced by Zadeh (1965) and this concept has found many application areas since then. For the first time Song and Chissom (1993a, b, 1994) introduced fuzzy time series. Fuzzy time series techniques proposed in literature generally consist of the three stages; these are fuzzification, determining fuzzy relations and defuzzification. In the literature, the decomposition of universe of discourse was mostly used in the fuzzification stage and intervals of it was determined arbitrarily as in Song and Chissom (1993a, b, 1994), Chen (1996, 2002). In addition, Huarng (2001a) has put forward the importance of the interval length on the forecasting performance and proposed two new techniques based on the mean and the distribution in order to find intervals. Egrioglu et al. (2010, 2011) have suggested forming the problem of finding intervals as an optimization problem. Chen and Chung (2006) and Lee et al. (2007) used the different interval lengths, instead of the fixed interval length, found by using the genetic algorithm and also Kuo et al. (2009, 2010), Davari et al.(2009), Park et al. (2010) and Hsu et al. (2010) used the particle swarm optimization.

The contribution of some studies in literature is to the stage of determining of fuzzy relations. In this stage, while Song and Chissom (1993a, b, 1994) used matrix operations, Chen (1996) and some others were using the fuzzy logic relations group table. Huarng and Yu (2006), Aladag et al. (2009) and Egrioglu et al. (2009a, b, c) used the artificial neural network for determining the fuzzy relations.

In the stage of defuzzification the centroid method is generally used in almost all studies except Song and Chissom (1994). Song and Chissom (1994) used artificial neural networks in the stage of defuzzification and also Yu (2005), Jilani et al. (2007), Jilani and Burney (2008) used different techniques in this stage.

In addition, other important studies on this issue have been suggested in the studies of Hwang et al. (1998), Huarng (2001b), Sullivan and Woodall (1994), Tsaur et al. (2005), Singh (2007), Cheng et al. (2008a, b).

In this study we used the differential evolution algorithm (DEA) in the fuzzification stage and fuzzy logic relations (FLR) and fuzzy logic group relations (FLGR) tables in the stage of the determination of fuzzy relations. We also consider the chronological-determined weights in the stage of defuzzification.

The remainder of this paper is organized as follows: The fundamental definitions of fuzzy time series are given in Section 2. DEA is briefly summarized in the Section 3. The proposed method is introduced in Section 4. The proposed method is supported by analysis of the time series data of student enrollments at University of Alabama between the years 1971 and 1992 and the results are evaluated in Section 5 and finally, Section 6 presents conclusions and discussions.

2. Fuzzy Time Series

The fuzzy time series was firstly defined by Song and Chissom (1993a). The basic definitions of fuzzy time series are given below;

Definition 1 Let $Y(t)\,(t=\cdots,0,1,2,\cdots)$, a subset of real numbers, be the universe of discourse by which fuzzy sets $f_i(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t),\cdots$ then $F(t)$ is called a fuzzy time series defined on Y(t).

Definition 2 Fuzzy time series relationships assume that F(t) is caused only by F(t-1), then the relationship can be expressed as: $F(t)=F(t-1)*R(t,t-1)$, which is the fuzzy relationship between F(t) and F(t-1), where * represents as an operator. To sum up, let $F(t-1)=A_i$ and $F(t)=A_j$ .The fuzzy logical relationship between F(t) and F(t-1) can be denoted as $A_i \rightarrow A_j$ where A_i (current state) refers to the left-hand side and A_j (next state) refers to the right-hand side of the fuzzy logical relationship. Furthermore, these fuzzy logical relationships can be grouped to establish different fuzzy relationship.

3. Differential Evaluation Algorithm

DEA was proposed by Price and Storn (1995). DEA is a heuristic algorithm based on the population. Mutation and cross-over operations are used in the process of DEA, respectively. All these operators are applied to all chromosomes in the population in DEA and a new chromosome is obtained. After using the mutation and cross-over operations the interested chromosome and the new chromosome are compared with a determined fitness value. Chromosome with a better fitness value is transferred to a new generation according to the objective function. The best chromosome is taken as the optimal solution at the end of the process of DEA. This information should be found enough in this section because DEA is discussed in more detail in the section of proposed method. Those who want more information can look the study of Price and Storn (1995).

4. Proposed Method

As it is well known that all stages of the fuzzy time series approaches influences very much intensively on the forecasting performance of the model. Many of studies in literature have contributed to first two stages, while few of them do to the defuzzification stage which is the last one. However this stage is effective as much as the others. Researchers have generally preferred to use the centroid method in the last stage so far.

In this study we used the DEA in the fuzzification stage and FLR and FLG tables in the stage of the determination of fuzzy relations. We also consider the chronological-determined weights in the stage of defuzzification. Briefly, we can summarize our proposed method as below.

Step 1 $D_{\min}$ and $D_{\max}$ are the minimum and maximum values of time series, respectively and the universe of discourse is defined in Equation 1.

$$U = [D_{lower} = D_{\min} - D_1,\ D_{upper} = D_{\max} + D_2] \quad (1)$$

D_1 and D_2 are the numbers determined arbitrarily.

If genes are shown with $x_i\,(i = 1,2,\cdots,(m-1))$ a chromosome structure of DEA can be shown as follows.

x_1	x_2	x_3	$\cdots$	x_{m-1}

Figure 1. A chromosome structure of DEA

According to the values of these genes, the parts of universe of discourse, i.e., sub-intervals are shown as follows:

$$u_1 = [D_{lower}, x_1], u_2 = [x_1, x_2], u_3 = [x_2, x_3], \cdots, u_m = [x_{m-1}, D_{upper}] \quad (2)$$

Step 2 The generation of initial population.

$x_{n,k,j}$ is to show the *n*. gene of *k*. chromosome of *j*. generation

$$x_{n,k,j=0} = D_{lower} + rand_n[0,1]*(D_{upper} - D_{lower}) \quad (3)$$

The genes produced by chromosomes are sorted by ascending order.

Step 3 For each chromosome in the population, the root of the mean squared error (RMSE) selected as the evaluation function is calculated by applying the steps from 3.1 to 3.5.

Step 3.1 m intervals based on the values of genes in chromosomes are used to form fuzzy sets as below.

$$A_i = a_{i1} / u_1 + a_{i2} / u_2 + \cdots + a_{im} / u_m,\ i = 1,2,\ldots m \quad (4)$$

here, a_{ik} is the membership degrees and it is shown in Equation 5.

$$a_{ik} = \begin{cases} 1 & ,k = i \\ 0.5 & ,\ k = i-1, i+1\ ,\ i = 1,2,\dots m \\ 0 & ,d.d. \end{cases} \tag{5}$$

The observations of crisp time series are converted to the fuzzy sets in which the interval in which the corresponding observation is included, has got the highest degrees of membership value.

Step 3.2 Obtain the FLRs and FLRG tables.

In our proposed method fuzzy relations are ordered by their chronologic occurrences and the weights are determined by chronologically and the last one has the biggest weight. The method is illustrated in Table 1.

Table 1. The weights chronologically determined

FLR	Weights
$A_1 \rightarrow A_1$	1
$A_1 \rightarrow A_2$	2
$A_1 \rightarrow A_1$	3
$A_1 \rightarrow A_1$	4

As we can easily understand from Table 1, we considered the chronologic order. This is based on the idea that the last occurred fuzzy relation should be largest effect on the forecast.

For example, when we observe the relation such as $F(t-1) = A_i$ and $F(t) = A_j$ for any time *t* this fuzzy logic relation is represented by $A_i \rightarrow A_j$.

Besides, the FLRs with the same Left Hand Side (LHSs) can be grouped into an ordered FLRG by putting all their RHSs together as on the Right Hand Side (RHS) of the FLRG. For example,

$A_i \rightarrow A_{j1}, A_i \rightarrow A_{j2}, \cdots, A_i \rightarrow A_{jk}$ can be grouped into an FLRG: $A_i \rightarrow A_{j1}, A_{j2}, \cdots, A_{jk}$

However, the repeated FLRs are counted only once.

Step 3.3 Obtain the fuzzy forecasts.

The fuzzy forecasts are obtained with respect to the fuzzy logic relation table formed in Step 3.2. For example; if $F(t-1) = A_i$ and there is a relation such as $A_i \rightarrow A_j$ in the fuzzy logic relation table then the fuzzy forecast will be A_j. If $F(t-1) = A_i$ and $A_i \rightarrow Empty$ in the fuzzy logic relation table then the fuzzy forecast will be A_i.

If $A_i \rightarrow A_{j1}, A_{j2}, \cdots, A_{jk}$ the forecast of F(t) is equal to $A_{j1}, A_{j2}, \cdots, A_{jk}$

Step 3.4 Defuzzify the fuzzy forecasts.

For example; If $F(t-1) = A_i$ and there exists the relation $A_i \rightarrow A_j$ in the table then the defuzzified forecast will be m_j which is the midpoint of u_j which is the subinterval of the fuzzy set A_j with the largest membership degree.

If $F(t-1) = A_i$ and there exists the relation $A_i \rightarrow Empty$ in the fuzzy logic relation table then the defuzzified forecast will be m_i which is the midpoint of the subinterval u_i which is the fuzzy set A_i with the largest membership degree.

Suppose the forecast of F(t) is $A_{j1}, A_{j2}, \cdots, A_{jk}$. The corresponding weights for $A_{j1}, A_{j2}, \cdots, A_{jk}$, say, $w_1, w_2, \cdots, w_k$ are specified. However, before forming the weight matrix with these $w_1, w_2, \cdots, w_k$ the weight matrix $W(t) = [w_1^{'}, w_2^{'}, \cdots, w_k^{'}]$ should satisfy a condition.

$$\sum_{h=1}^{k} w_h^{'} = 1 \tag{6}$$

Hence, these weights $w_1, w_2, \cdots, w_k$ should be standardized. We then obtain the following weight matrix:

$$W(t) = [w_1^{'}, w_2^{'}, \cdots, w_k^{'}] = \left[\frac{w_1}{\sum_{h=1}^{k} w_h}, \frac{w_2}{\sum_{h=1}^{k} w_h}, \cdots, \frac{w_k}{\sum_{h=1}^{k} w_h} \right] \tag{7}$$

where w_h is the corresponding weight for A_{jh}:

$$W(t) = [w_1^{'}, w_2^{'}, \cdots, w_k^{'}] = \left[\frac{w_1}{\sum_{h=1}^{k} w_h}, \frac{w_2}{\sum_{h=1}^{k} w_h}, \cdots, \frac{w_k}{\sum_{h=1}^{k} w_h} \right] = \frac{1}{\sum_{h=1}^{k} w_h}, \frac{2}{\sum_{h=1}^{k} w_h}, \cdots, \frac{k}{\sum_{h=1}^{k} w_h} \tag{8}$$

For example, when the forecast of F(t) is A_1, A_2, A_1, A_1 the weights are specified as follows:

$w_1 = 1, w_2 = 2, w_3 = 3, w_4 = 4$ the weight matrix is determined as

$$W(t) = \left[\frac{1}{1+2+3+4}, \frac{2}{1+2+3+4}, \frac{3}{1+2+3+4}, \frac{4}{1+2+3+4} \right]$$

$$W(t) = \left[\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10} \right]$$

Then, the defuzzified forecast is calculated as below (Yu (2005)).

$$\hat{x}_t = \sum_{h=1}^{k} m_h w_h^{'} \tag{9}$$

Step 3.5 Let x_t be the original time series and $\hat{x}_t$ be its defuzzified forecasts with the n observations. RMSE is calculated by

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} (x_t - \hat{x}_t)^2}{n}} \tag{10}$$

Step 4 Mutation and Cross-over operations.

Mutation and Cross-over operations are applied to for each chromosome in the initial population, respectively.

Step 4.1 The application of Mutation operation

Generally, for applying mutation operation in genetic algorithm, a chromosome is selected randomly and a random gene of this chromosome is changed with a new gene generated randomly.

Applying mutation operation in DEA, three different chromosomes except interested chromosome are selected.

The first two of these selected three chromosomes are subtracted each other and it is called as the difference vector. Then, difference vector is multiply by F and a new chromosome is obtained (In general the parameter *F* gets values between 0 and 2. We take this *F* value as 0.8 which is general value in the literature in this study). This new chromosome is called as the weighted difference vector. The weighted difference vector is summed with the third chromosome and mutation is completed. This new created chromosome is called as the total vector.

Thus, the chromosome to be used in the cross-over operation is created with the help of mutation operation.

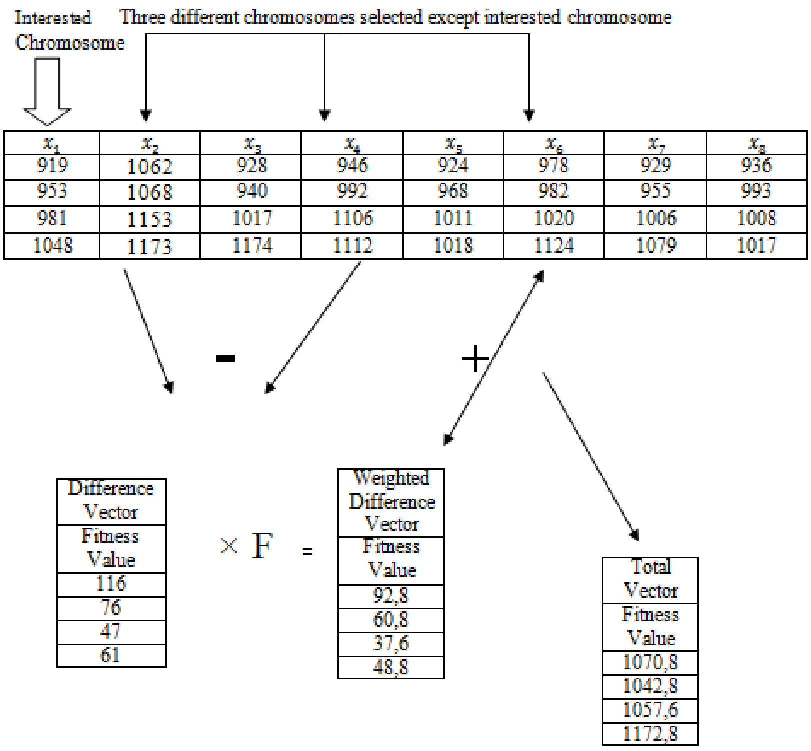

Figure 2. An example of Mutation operation

Step 4.2 The application of Cross-over.

Applying Cross-over operation, the total vector obtained from at the end of the mutation operation is compared with interested chromosome and nominee chromosome is obtained.

While nominee chromosome is obtained each gene of total vector and interested chromosome is evaluated one by one.

First at all, a cross-over rate *(cor)* is determined. Then, a random number is generated between 0 and 1 with the help of uniform distribution.

If this random number is smaller than the cross-over rate, the gene is taken from total vector. If it is not, the gene is taken from interested vector and nominee chromosome is generated and the fitness value of nominee chromosome is calculated.

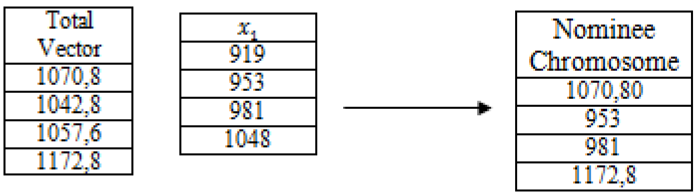

Figure 3. An example of Cross-over operation

Step 5 The comparison of fitness values

Nominee chromosome and interested chromosome are compared in terms of fitness values. The chromosome which has the smaller RMSE value used as evaluation function is transferred to new generation.

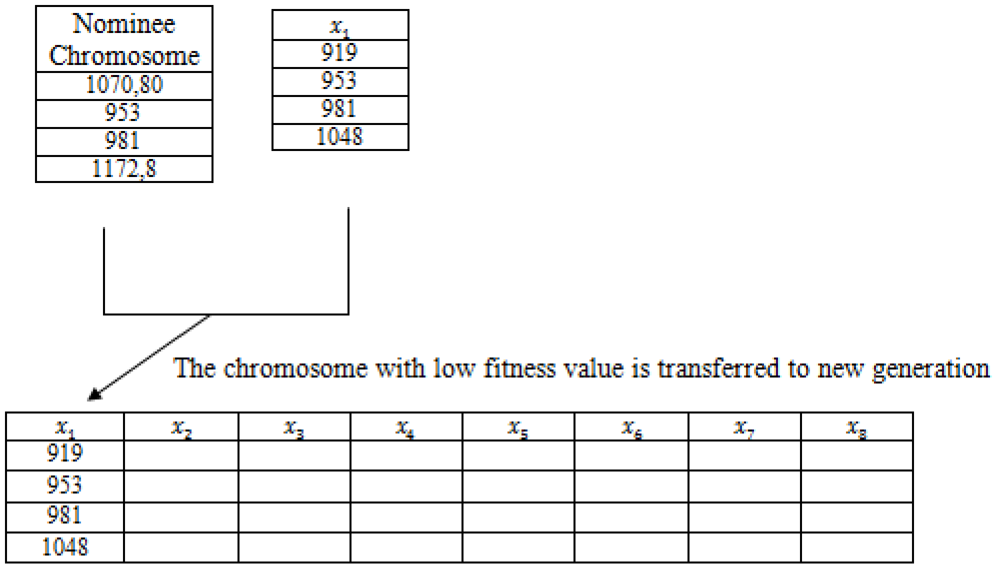

Figure 4. An example of creation of a new generation

All these operations are applied to each chromosome in initial population individually.

Step 6 Steps 3-5 are repeated as much as a predetermined number of iterations.

5. Application

In order to show the performance of the proposed method, the proposed method is applied to the time series data of student enrollments at University of Alabama between the years 1971 and 1992 which is frequently used in the literature. The results are compared with the results from some methods which are already in fuzzy time series literature.

The universe of discourse is determined in accordance with data as $U = [13000, 20000]$ in analysis.

In the proposed method, 1600 different solutions with the following properties were obtained.

- The number of chromosomes *(cn)* was tested between 10 and 100 with increment 10 as totally10 alternatives.
- Cross-over rate *(cor)* was tested between 0.1 and 1 with increment 0.1 as totally 10 alternatives.

- The intervals *(m)* were tested between 5 and 20 with increment 1 as totally 16 alternatives.

We conclude that the best result is obtained in the case where *cn=80, cor=1, m=20*. Table 2 presents RMSE values obtained from the proposed method and some other methods proposed in literature, comparatively.

Table 2. A comparative presentation of enrollments' RMSE values by various methods

Methods	RMSE
Song and Chissom (1993a)	642
Song and Chissom (1993b)	650
Song and Chissom (1994)	880
Sullivan and Woodall (1994)	621
Chen (1996)	638
Hwang et al. (1998)	528
Huarng (2001a)	353
Huarng (2001b)	476
Tsaur et al. (2005)	367
Singh (2007)	365
Cheng et al. (2008)	438
Cheng et al. (2008)	478
Proposed Method	**326**

Enrollment data and the forecasts obtained from proposed method are given in Table 3.

Table 3. Enrollment data and the forecasts obtained from proposed method

Year	Enrollment Data	Forecasts	Year	Enrollment Data	Forecasts
1971	13055	-	1982	15433	15411
1972	13563	13527	1983	15497	15535
1973	13867	13841	1984	15145	15535
1974	14696	14469	1985	15163	15535
1975	15460	15411	1986	15984	15467
1976	15311	15523	1987	16859	16772
1977	15603	15523	1988	18150	17924
1978	15861	15501	1989	18970	18552
1979	16807	16877	1990	19328	18866
1980	16919	17296	1991	19337	18448
1981	16388	16354	1992	18876	18657

6. Conclusions and Discussions

It has been observed that there exist many factors which affect the forecasting performance of fuzzy time series approach recently considered by researchers. In this study we used the DEA in the fuzzification stage and FLR and FLGR tables in the stage of the determination of fuzzy relations. We also consider the chronological-determined weights in the stage of defuzzification. This is based on the idea that the last occurred fuzzy relation should be largest effect on the forecast.

We expect that in future studies researchers can concentrate on a different optimization technique for finding the weights used in the defuzzification stage.

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