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MATHEMATICAL AND ECONOMICAL MODELING OF SUPPLY CHAIN, CASE STUDY: DAIRY RIO – ALGERIA

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Abstract

In this article, we will try to show the effective role played by the forecast methods of sales to manage an extremely important function in the firm which is the supply chain management. However, we point the way to use the data of forecasting in the mathematical modeling for the supply chain which is distinguished by the multi purposes objectives. Modeling can be done by using different models such as goal programming model with its different variants that can be adapted to the case studied, and referring to the work of specialists in the field (Ignizio JP (1982), Charnes A . and WW Cooper (1961), Martel and JM Aouni B. (1990)), the method of Box and Jenkins, mathematical programming with multiple objectives etc. Modeling will be done by using the method of Lot-sizing while the resolution will be through the use of multi-criteria methods. In our particular case, we will try to apply them to the "RIO firm" an important Algerian firm in producing yogurt. These methods and results will be used by the "Rio company "officials in their strategic management and mainly the supply chain of products.

Keywords: forecast, sales, supply chain, mathematical modeling, lot-sizing models, multi criteria methods

JEL Classification: C44, C53, C54, C61, D24

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1. Introduction

Revealed by the old economy for several years, confirmed by the new economy, the logistics proved to be not only the heart of enterprises strategies, but also to the very root of their performance in carrying out daily operational tasks.

It is above all thanks to her that strategies are made possible or fail. Logistics is complex because it is diffuse and multifaceted (Paché and Sauvage, 2004).

The nature of the products, company history, its legacy, and its current markets, are the determining factors for the strategies chosen to retain lead logistics solutions (Giard, 2003).

This latter is based mainly on sales forecasting and integration of its components seeking to reduce the time needed for the supply process, stock levels early in the beginning of each season, as well as reducing risks to meet customer demand.

To achieve these objectives it is necessary to use a comprehensive package of technical methods such as sales forecasting and multi criteria methods applied to an Algerian company.

2. Theoretical section

2.1 Definition of supply chain management

Supply chain management is defined as the process responsible for managing and developing the total logistics system of the organization with its internal and external components. At operational levels, it includes and overtakes the purchasing and supply activities, adding many aspects of strategic focus (Samii, 2004).

There are many definitions of supply chain management which lead scholars to try to find a single definition. Such a definition is supposed to contain the main components of supply chain management. These components are: (Stadtler and Kilger, 2000, 2002; Marchal, 2006):

- The group targeted,
- The objectives,
- Appropriate means to achieve these objectives.

2.2 Definition of sales forecasting

Sales forecasting is an attempt to estimate the level of future sales through the use of previous and current information available about the phenomenon under study (sales). It is an attempt by the company to know the future based on past and present. Definitely; this does not lead to a precise calculation predicting the future but it helps to estimate the future through using technical and scientific methods (Bourbonnais and Usunier, 2004).

The forecast is a series of calculations used to estimate future; it combines art, science and individual contributions for the study and determination of the assumptions on which the forecast is made. This is so important knowing that the forecasting is a key behavioral indicator of business administration once making future plans (Doriath and Gouget, 2002).

2.3 The role of sales forecasting in the supply chains management

Sales forecasting plays an important role in supply chain management. Arguably the latter is responsible for the strategic management of material and products flows within and outside the company as well as their stocking (Breuzard and Fromentin, 2004).

On the second partial process of supply chains, its purpose is to implement the needed plans to achieve integration between the company’s different activities (Pimor, 2005).

Accordingly, the sales forecasting is considered as an essential and indispensable basis for the management of supply chains.

2.4 The multi criteria formulation of a decision problem

In the terminology introduced by Vansnick (1990), the formulation of a multi criteria decision problem can be defined as the model "A, A / F, E" where (Charnes and Cooper, 1961):

A is the set of potential actions (feasible, permissible). This set can be explicitly defined (finite set), the constraints are implicit or explicit. .

In this second case, one uses the multiple objectives mathematical programming (MOMP) and often referred all eligible shares by the symbol X;

A / F is the finite set of attributes or criteria , usually conflicting, from which the shares will be valued, and E is the set of performance evaluations of the alternatives for each attribute or criteria, that is to say the set of performance vectors, one vector per share.

2.5 Methods of sales forecasting and modeling of supply chains

There are several methods of forecasting sales that vary in their ease of application and the accuracy of their results.

There exist such simple and easy qualitative methods; which do not require much skill and experience. These methods are based mainly on intuitive perception and the induction of future imagery from statistical data. Other methods are based on market research by using the method of treating a range of sampling to determine consumer demand forecasting based on field experience. The shortcomings of these methods lie in the fact that they are based on intuition and conjecture.

Quantitative methods using econometric techniques are also used for understanding the behavior of certain variables in the past and predict their behavior in the future such as the exponential smoothing method, the method of Box and Jenkins, the goal programming, compromise the programming, programming using the functions of satisfaction etc.

We use the following three methods: the method of Box and Jenkins (sales forecasting), Lot-sizing models (modeling of supply chains) and the method of compromise programming (resoling the model)

2.5.1 The method of Box and Jenkins

Box and Jenkins (1976) proposed a prediction technique for univariate series based on the notion of ARIMA process. This technique has three steps: identification, estimation and verification. This method is used to obtain a model explaining the fluctuations of a series based solely on the past conduct and then extrapolate the values of the variable. If the series suggests a pattern that repeats fairly regularly, the choice of this method makes sense (Eric Dor, 2004; Bourbonnais and Terraza, 1998).

- The first step is to identify the ARIMA (p, d, q) which could cause the series. The series should be transformed first to make it stationary and then identify the ARMA (p, d)

- The second step is to estimate the ARIMA model using a nonlinear method (nonlinear least squares or maximum likelihood).

- The third step is to check whether the estimated model reproduces the model that generated the data.

Any stationary process can be approximated by models AR (p), MA (q) or ARMA (p, q).

1) AR(p) : Autoregressive models of order p.

The autoregressive part of a process (AR) consists of a finite linear combination of previous values of the process. The AR (p) is defined by the following general formula:

$$x_t = a_1 x_{t-1} + a_2 x_{t-2} + ... + a_p x_{t-p} + \varepsilon_t$$

With a Gaussian white noise process, for example:

$$AR(1): \quad x_t = a_1 x_{t-1} + \varepsilon_t$$

$$AR(2): \quad x_t = a_1 x_{t-1} + a_2 x_{t-2} + \varepsilon_t \quad etc...$$

2) MA(q) : Models based on moving averages.

The moving average part is made up of a finite linear combination of previous values at t of a white noise. The MA (q) is defined using the following formula:

$$MA(q): \quad x_t = \varepsilon_t - b_1 \varepsilon_{t-1} - b_2 \varepsilon_{t-2} - ... - b_q \varepsilon_{t-q}$$

With a Gaussian white noise process, for example:

$$MA(1): \quad x_t = \varepsilon_t - b_1 \varepsilon_{t-1}$$

$$MA(2): \quad x_t = \varepsilon_t - b_1 \varepsilon_{t-1} - b_2 \varepsilon_{t-2} \quad etc...$$

3) ARMA models.

These models are based on a combination of both previous models (AR and MA) and are representative of a process generated by a combination of previous values and errors. They are defined by the following general formula:

$$ARMA(p,q): \quad x_t = a_1 x_{t-1} + a_2 x_{t-2} + ... + a_p x_{t-p} + \varepsilon_t - b_1 \varepsilon_{t-1} - b_2 \varepsilon_{t-2} - ... - b_q \varepsilon_{t-q}$$

With a Gaussian white noise process, for example:

$$ARMA(1,1): \quad x_t = a_1 x_{t-1} + \varepsilon_t - b_1 \varepsilon_{t-1}$$

$$ARMA(2,1): \quad x_t = a_1 x_{t-1} + a_2 x_{t-2} + \varepsilon_t - b_1 \varepsilon_{t-1} \quad etc...$$

The AR, MA and ARMA are representative of that chronic

- Stationary average

- Seasonally adjusted values.

ARIMA models and SARIMA take into account the stationary of the series which are the basis of estimations; this stationary generates either the trend (ARIMA) or a problem related to seasonality (SARIMA).

There are several tests which can help us choosing the most appropriate model such as the Akaike test (1969), the Schwar test (1978), the Hannan-Quinn test (1979), etc.

The choice of model will be based on the smallest value of one of these tests.

2.5.2 Lot-sizing models

The Lot-sizing models are deterministic mathematical models for the medium-term planning which consist the basis of our proposal. They are extremely numerous, but can be classified according to various criteria (Fig.1) (Thierry , 2003):

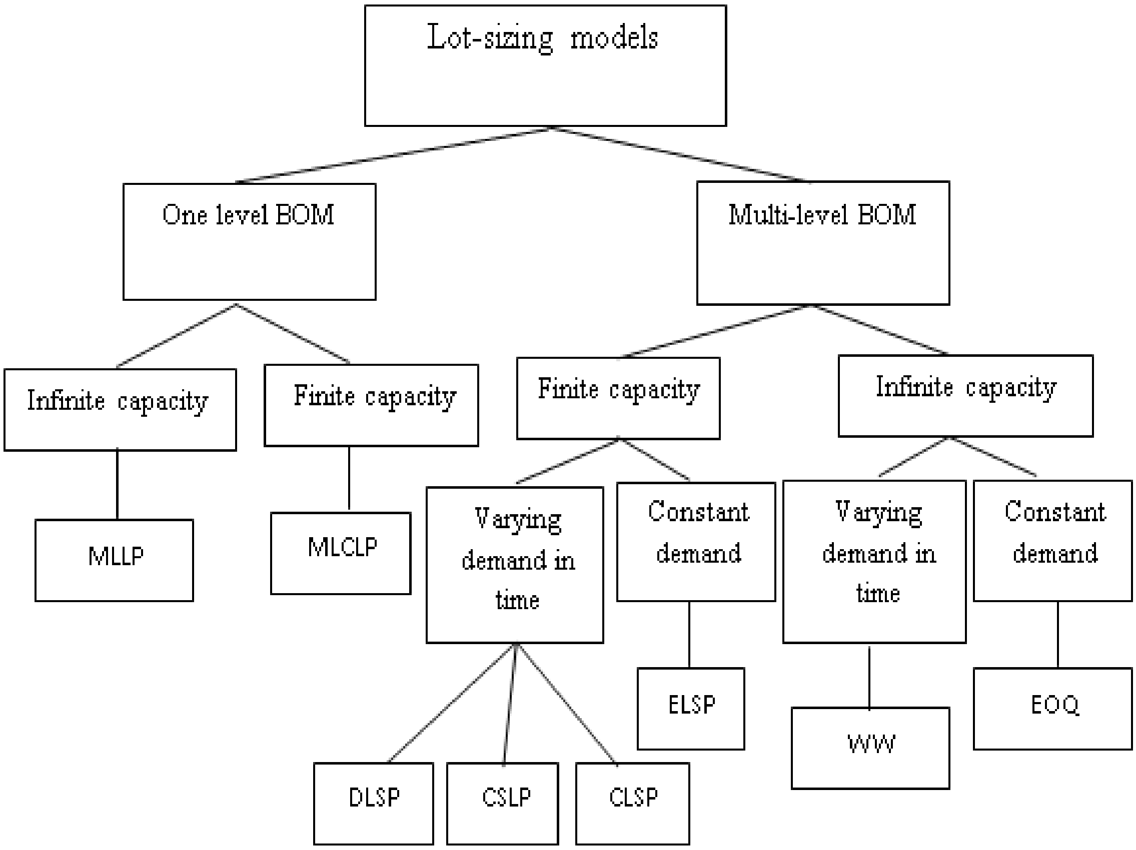

Figure 1. Lot-sizing models

EOQ : Economic order quantity.

WW : Wagner-Whitin.

ELSP: Economic Lot-sizing and Scheduling problem.

CLSP: Capacitated Lot-Sizing Problem.

CSLP: Continuous Set-up Lot-sizing Problem.

DLSP: Discrete Lot-sizing and Scheduling Problem.

MLLP: Multi-Level Lo-sizing Problem.

MLCLP: Multi-Level Capacitated Lot-sizing Problem.

Using different mathematical modeling techniques based mainly on models of Lot-sizing, we analyze several new decision variables and the mathematical formulation of the model we will use the case study is as follows:

- The objectives of the supply chain:

$$1: Min\left[\sum_{i\in N}\sum_{t\in T}\left[\begin{array}{l}\alpha(i)h_i I_{i,t}+\sum_{k\in K}\beta(i)p_{i,t}X_{i,k,t}\\+\gamma(i)DAp_{j,t}\times CAp_j+\theta(i)D_{i,t}\times CD_i\end{array}\right]\right]$$

$$2: Max\left[\sum_{i\in N}\sum_{t\in T}a_{i,t}D_{i,t}\right]$$

Given that the objective (1) is the minimization of costs of the supply chain management (procurement, storage, production and distribution). The objective (2) is to maximize the profit during the planning period.

With:

T: all periods of the planning horizon;

N: set of products (finished products, components, raw materials);

K: set of resources;

h_i : Storage cost of product i (which may be a raw material, components or finished products);

$p_{i,t}$: Cost of producing one unit of product i in period (t);

CAp_j : Supply cost of one unit of j;

CD_i : Distribution cost of one unit of i;

$a_{i,t}$: Profit of the distribution one unit of product i in period (t);

The decision variables of the model are related to:

Production: $X_{i,k,t}$ represents the quantity of product i produced on the resource k in period t;

Supplies: $DAP_{i,t}$ represents the amount of raw material i that must supply during the period t;

Distribution: $D_{i,t}$, represents the quantity of finished product i distributed in t;

And finally, the state variable $I_{i,t}$ is the stock level of product i at the beginning of t.

Constraints: The constraints of the planning model for supply chain management are:

- The evolution of the stock:

$$I_{i,t+1}=I_{i,t}+\sum X_{i,t,k}-\sum g_{i,j}X_{i,t,k}+DAp_{i,t}-D_{i,t} \text{..(1)}$$

- Limited capacity:

* Production:

$$\sum_{i \in N} b_{i,k} X_{i,k,t} \leq C_{k,t} \,/\, k \in K, t \in T ....(2)$$

* Storage:

$$\sum_{i \in N} I_{i,t} \leq S_t \,/\, t \in T ....(3)$$

* Transportation:

$$\sum_{i \in N} D_{i,t} \leq T_t \,/\, t \in T ....(4)$$

$$X_{i,k,t}, DAp_{i,t}, D_{i,t}, I_{i,t} \geq 0 ....(5)$$

The constraint (1) calculates the stock level change between two consecutive periods. This equation, called state equation, involves the production achieved according to the nomenclature of the product over the period, the quantities that should be expected supplies and deliveries. Constraints (2, 3, 4) can limit the production, transport and storage according to the available capacity. Finally, constraint (5) indicates that all decision variables are nonnegative.

With:

$g_{i,j}$: Quantity of product j required to produce one unit of product i;

$b_{i,k}$: Amount of resources required to manufacture one unit of product i on resource k (Range of manufacture);

$C_{k,t}$: Production capacity of resource k in period t.

2.5.3 The compromise programming

This model aims to solve the economic problems which have conflict objectives whose optimal solutions are not known. The analytical form of this model is as follows (Ignizio, 1982):

$$g_i = \begin{cases} g_i^* = Max \quad f_i(x), x \in F \\ g_i^* = Min \quad f_i(x), x \in F \end{cases}$$
$$C_l(x) \le 0, \quad l = 1, 2, ..., L$$

With:

g_i: functions of goals.

$f_i(x)$: linear functions.

In this model we have two goals $f_1(x)$ et $f_2(x)$ and where we want to maximize the first and minimize the second under the constraints. $C_l(x)$.

Solving this kind of model involves two important steps:

- Seek the maximum or minimum value of each objective separately constrained by the use of linear programming.

- Solve the model by the use of weighted goal programming where we give a weight (w_i) to each objective as follows:

$$Min \quad Z = \sum_{i=1}^{m} w_i (\delta_i^+ + \delta_i^-)$$
$$sujet \quad à$$
$$\begin{cases} f_i(x_j) + \delta_i^- - \delta_i^+ = g_i^* \\ f_i(x_j) + \delta_i^- - \delta_i^+ = g_i^* \\ C_l(x) \le 0, \quad l = 1, 2, ..., L \\ x_i \le 0 \quad avec \quad i = \{1, 2, 3, ..., n\} \end{cases}$$

With:
i: the number of the objective function.
j: the number of decision variable.
l: the number of constraints.

And δ_i^+ et δ_i^- the positive and negative deviations for the gap between the level of achievement of the goal and aspiration level (of the decision maker).

3. Case study of the dairy Rio (mathematical modeling of supply chains for products)

The case study will involve an Algerian company, the dairy Rio.

3.1 Presentation of the dairy

The dairy Rio is a limited liability company since June 2004 with a capital estimated at 500,000 Dinars. It was established in 1999 under a family character. This company is specialized in the production of yoghurt and employs 26 workers (11 men and 15 women). It is localized in Tlemcen city in the north-western part of Algeria, where it conducts its production activity.

3.2 The current situation of the dairy Rio

The company faced a particular situation in recent years for the following reasons:

- The entry of more competitors in the production of yoghurt at low prices.

- The deterioration of the company's products quality might lead to stop the production in this company. Thereby the company has chosen to specialize in the production of three types of yoghurt: Stirred yoghurt, flavored yoghurt packaged in tonic jars and flavored yoghurt packaged in simple plastic jars.

It becomes difficult to determine the proper method of predicting if we do not know the nature of the product and the forecast period. In addition, the modeling process cannot be achieved without knowing the various objectives and constraints imposed by determinants of products such as the time required for procurement, production and distribution as well as the capacity available to the company.

After a detailed study of the products' characteristics of this company, we have compiled the following table:

Table 1. Product features of the dairy Rio

Product	**Flavored yoghurt packaged in simple plastic jars**	**Flavored yoghurt packaged in TONIC jars**	**Stirred yoghurt with fruit packed in TONIC jars**
Production time of 2300 jars	25 h et 45 m	26 h et 15 m	27 h et 15 m
Selling price of one (1) jar (DA)	36	40	52
Storage cost per unit (DA)	2	2	2
Distribution cost per unit (DA)	0.58	0.58	0.58
Cost of one (1) jar (DA)	28.10	30.97	41.04
Unit profit (DA)	5.32	6.45	8.38
Product Quality	acceptable	good	very good
expiry run times	30 days	30 days	30 days

Fixed costs for managing the supply chain have been estimated at 8220 DA.

3.3 The problem in supply chain management of the dairy

We know that the management objectives of the supply chain are summarized in meeting customer needs in terms of quantity needed, at the right time, in the right place and at an acceptable quality. All this must happen for the company that will minimize costs and maximize profit.

Through the study of the products characteristics of the dairy Rio, we noticed that there were constraints that limit the achievement of these goals. One can cite in this sense the short life of these products which does not exceed 30 days; the company's customers are retailers, which require very short-term forecasts (weekly sales forecasting). These points led us to study the weekly sales of dairy products during the years 2007 and 2008 to carry out their modeling and forecasting.

We will also consider the constraint of limited daily production of these products and the time needed to produce more food difficulties due to different causes. The process of achieving the objectives of supply chain management (relatively) requires the use of a multicriteria method called "compromise programming."

As for the objectives in this case study, they are summarized as follows: to maximize the quality of overall sales, minimize costs in the supply chain, maximize the company's profit and meet the customers' demand forecasting.

3.4 Forecasting sales of the first week of 2009 for the three products.

We used the method of Box and Jenkins to forecast weekly sales of products.

It has established the following forecasting models using the software Eviews 5.1:

- Flavored yoghurt packaged in TONIC jars:

$$\Delta YEFT_t = -0.86\varepsilon_{t-1} + \varepsilon_t$$

$YEFT_t$: Sales of Stirred yoghurt at time t.

$$\Delta YEFT_t = YEFT_t - YEFT_{t-1}$$

ε_t: white noise at time t

-Flavored yogurt packaged in jars TONIC

$$YEAT_t = 1.007 YEAT_{t-1} - 0.99\varepsilon_{t-1} + \varepsilon_t$$

$YEAT_t$: Sales of Flavored yoghurt packaged in simple plastic jars at time t.

-Flavored yogurt packaged in simple plastic pots

$$\Delta YEAS_t = -0.71\varepsilon_{t-1} + \varepsilon_t$$

$YEAS_t$: Sales of flavored yogurt packaged in simple plastic pots at time t

$$\Delta YEAS_t = YEAS_t - YEAS_{t-1}$$

We note that the time series of weekly sales of three products are first degree stationary; knowing that the series of sales of stirred yoghurt fruit is affected by random error of the previous period which may have occurred during a period and acts on these values. As for the series of sales of flavored yoghurt packaged in TONIC jars, it is affected by its previous value and the random error of the previous period, while the series of sales of flavored yoghurt packaged in simple plastic jars is affected by random error of the previous period.

Predictable sales results of the first week of January 2009 are:

Table 2. Sales expected in the 4 first weeks of 2009

Products	Flavored yoghurt packaged in simple plastic jars	Flavored yoghurt packaged in TONIC jars	Stirred yoghurt with fruit packed in TONIC jars
sales expected of the first week of January	17459	4164	11177
sales expected of the second week of January	17459	4193	11177
sales expected of the third week of January	17459	4222	11177
sales expected of the fourth week of January	17459	4252	11177

Source: Table developed by researchers on the basis of previous models

3.5 Mathematical modeling of the supply chain

The problem facing the management of the supply chain of products of the dairy Rio is represented by how determined is the quantity produced and when it must be produced to achieve the Company' supply chain Management objectives; which can be cited as the following ones:

- Minimization of the supply chain total costs.
- Maximization of total profit.
- Optimizing the products quality.

Quality was assessed as very good: 16 / 20, good: 14 / 20, acceptable: 12 / 20

$$Z_1 Min = \begin{bmatrix} 2\sum_{i=1}^{3}\sum_{t=1}^{4} I_{it} + 41.04\sum_{t=1}^{4} X_{1t} + 30.97\sum_{t=1}^{4} X_{2t} + 28.1\sum_{t=1}^{4} X_{3t} + 0.38\sum_{t=1}^{4} DAp_{1t} + \\ 0.044\sum_{t=1}^{4} DAp_{2t} + 6\sum_{t=1}^{4} DAp_{3t} + 0.6\sum_{t=1}^{5} DAp_{4t} + 0.17\sum_{t=1}^{4} DAp_{5t} + \\ 0.58\sum_{i=1}^{3}\sum_{t=1}^{4} D_{it} + 8220 \end{bmatrix} \ldots(1)$$

$$Z_2 Max = \left[8.38\sum_{t=1}^{4} D_{1t} + 6.45\sum_{t=1}^{4} D_{2t} + 5.32\sum_{t=1}^{4} D_{3t} - 8220 \right] \ldots\ldots(2)$$

$$Z_3 Max = \left[16\sum_{t=1}^{4} D_{1t} + 14\sum_{t=1}^{4} D_{2t} + 12\sum_{t=1}^{4} D_{3t} \right] \ldots\ldots(3)$$

x_{1t}: the quantity produced of stirred yoghurt with fruit packed in TONIC jars.
x_{2t} : the quantity produced of flavored yoghurt packaged in TONIC jars.
x_{3t} : the quantity produced of flavored yoghurt packaged in simple plastic jars.

Nevertheless; there are several constraints or objective conditions that limit the achievement of optimum level of these objectives and which are represented by:

- The volume of production that do not exceed the volume of sales projections for the 4 first weeks of January 2009.

- Stress-generating capacity and characteristics of products.

- The weekly volume of production of stirred yoghurt with fruit packed in TONIC jars must exceed 5000, the flavored yoghurt packaged in TONIC jars 2000 while the volume of production of flavored yoghurt packaged in simple plastic jars must exceed 9000 jars because weekly demand for three products during the last weeks of 2008 has not dropped below these amounts.

- The stress of weekly hours available: the number of hours available weekly production was estimated at about 2160 minutes (Thursday, Saturday, Sunday and Monday) the calculation of time spent in producing one unit of the three products presents the following results:

x1: 0.124 mn ; x2: 0.098 mn ; ; x3: 0.085 mn.

$$I_{i,t+1} = I_{i,t} + X_{it} - D_{it} \,/\, i = \{1,2,3\}, t = \{1,2,3,4\}$$
$$D_{1t} \geq 5000 \,/\, t = \{1,2,3,4\}$$
$$D_{2t} \geq 2000 \,/\, t = \{1,2,3,4\}$$
$$D_{3t} \geq 9000 \,/\, t = \{1,2,3,4\}$$
$$DAp_{jt} = \sum_{i=1}^{3} g_{ij} X_{it} \,/\, j = \{1,2,3,4,5\}$$
$$5000 \leq I_{1t} + X_{1t} \leq 11177 \,/\, t = \{1,2,3,4\}$$
$$2000 \leq I_{21} + X_{21} \leq 4164$$
$$2000 \leq I_{22} + X_{22} \leq 4193$$
$$2000 \leq I_{23} + X_{23} \leq 4222$$
$$2000 \leq I_{24} + X_{24} \leq 4252$$
$$9000 \leq I_{3t} + X_{3t} \leq 17459 \,/\, t = \{1,2,3,4\}$$
$$X_{1t} + X_{2t} + X_{3t} \leq 27600 \,/\, t = \{1,2,3,4\}$$
$$0.124 X_{1t} + 0.098 X_{2t} + 0.085 X_{3t} \leq 2160 \,/\, t = \{1,2,3,4\}$$
$$X_{i,k,t}, DAp_{i,t}, D_{i,t}, I_{i,t} \geq 0$$

3.6 Resolution of the model using the method of compromise programming

The final mathematical form of the model takes the following form:

$$\text{Min Z} = \delta_1^+ + 4\delta_2^- + 2\delta_3^-$$

Under the constraints:

$$\left[\begin{array}{l} 2\sum_{i=1}^{3}\sum_{t=1}^{4} I_{it} + 41.04\sum_{t=1}^{4} X_{1t} + 30.97\sum_{t=1}^{4} X_{2t} + 28.1\sum_{t=1}^{4} X_{3t} + 0.38\sum_{t=1}^{4} DAp_{1t} + \\ 0.044\sum_{t=1}^{4} DAp_{2t} + 6\sum_{t=1}^{4} DAp_{3t} + 0.6\sum_{t=1}^{5} DAp_{4t} + 0.17\sum_{t=1}^{4} DAp_{5t} + \\ 0.58\sum_{i=1}^{3}\sum_{t=1}^{4} D_{it} + \delta_1^- - \delta_1^+ + 8220 \end{array}\right] = 3307772$$

$$\left[8.38\sum_{t=1}^{4} D_{1t} + 6.45\sum_{t=1}^{4} D_{2t} + 5.32\sum_{t=1}^{4} D_{3t} + \delta_2^- + \delta_2^+ - 8220 \right] = 559016.8$$

$$\left[16\sum_{t=1}^{4} D_{1t} + 14\sum_{t=1}^{4} D_{2t} + 12\sum_{t=1}^{4} D_{3t} + \delta_3^- + \delta_3^+ \right] = 1192419$$

$$
\begin{aligned}
& I_{i,t+1} = I_{i,t} + X_{it} - D_{it} \,/\, i = \{1,2,3\}, t = \{1,2,3,4\} \\
& D_{1t} \geq 5000 \,/\, t = \{1,2,3,4\} \\
& D_{2t} \geq 2000 \,/\, t = \{1,2,3,4\} \\
& D_{3t} \geq 9000 \,/\, t = \{1,2,3,4\} \\
& DAp_{jt} = \sum_{i=1}^{3} g_{ij} X_{it} \,/\, j = \{1,2,3,4,5\} \\
& 5000 \leq I_{1t} + X_{1t} \leq 11177 \,/\, t = \{1,2,3,4\} \\
& 2000 \leq I_{21} + X_{21} \leq 4164 \\
& 2000 \leq I_{22} + X_{22} \leq 4193 \\
& 2000 \leq I_{23} + X_{23} \leq 4222 \\
& 2000 \leq I_{24} + X_{24} \leq 4252 \\
& 9000 \leq I_{3t} + X_{3t} \leq 17459 \,/\, t = \{1,2,3,4\} \\
& X_{1t} + X_{2t} + X_{3t} \leq 27600 \,/\, t = \{1,2,3,4\} \\
& 0.124 X_{1t} + 0.098 X_{2t} + 0.085 X_{3t} \leq 2160 \,/\, t = \{1,2,3,4\} \\
& X_{i,k,t}, DAp_{i,t}, D_{i,t}, I_{i,t} \geq 0
\end{aligned}
$$

δ_1^- et δ_1^+ : negative and positive deviations of costs achieved in relation to its minimum level.

δ_2^- et δ_2^+ : negative and positive deviations of the profit achieved in relation to its maximum level.

δ_3^- et δ_3^+ : negative and positive deviations of the quality achieved in relation to its maximum level

Through using the software Lindo61, we obtained the following optimal solution:

$$Z_1 Min = 4480275.3$$
$$Z_2 Max = 550250.96$$
$$Z_3 Max = 1192419$$
$$X_{11} = X_{12} = X_{13} = X_{14} = 5000$$
$$X_{21} = 4164$$
$$X_{22} = 4193$$
$$X_{23} = 4222$$
$$X_{24} = 4252$$
$$X_{31} = 13316$$
$$X_{32} = 13283$$
$$X_{33} = 13294$$
$$X_{34} = 13215$$

The results can be interpreted as follows.The dairy Rio must produce 5,000 jars of fruit yogurt stirred in each week of the scheduled period and 4164, 4193, 4222 and 4252 flavored yogurt packaged in jars TONIC in the 1st, 2nd, 3rd and 4th week in successively and 13316, 13283, 13294 and 13215 flavored yogurt packaged in simple plastic pots in the 1st, 2nd, 3rd and 4th week in succession. And the stock of the beginning of each week must be zero. This production requires an amount of DA 4480275.3 representing the cost of managing the supply chain and realize the maximum profit estimated at 550 250.96 DA and a better quality of sales. However, we note that these results through the dairy Rio does not meet all the demand and therefore, it is for the company to expand its production capacity.

4. Conclusion

In this research we tried to show how to use methods of sales forecasting and modeling as a strategic tool for supply chain management; which is considered as a useful technology in channeling the flow of raw materials, semi-finished and finished products, from the first suppliers to final customers at reduced costs, depending on the quantities requested, where and when appropriate. At each step of the planning process, the manager must make the best decision among a broad set of available alternatives. However; this is done in order to ensure proper management of logistics including all kinds of materials, either inside or outside the company, in ways to minimize costs and time as well as to raise the service level.

The Case Study focused on the dairy Rio has shown that this company is experiencing difficulties in recent years because of the increase in competition which is getting fiercer every year. Analysis of its sales occurred over two years (2007 and 2008), in order to know the historical and explanatory factors. Three important goals were to achieve: maximizing the profit, minimizing costs and maximizing the quality of products. This modeling allowed us to release the amount to be produced by the dairy in order to achieve the objectives of the supply chain management.

However, these techniques and methods are tools to facilitate decision making and must be supplemented by the experience and expertise of the decision makers themselves.

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