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A SEASONAL FUZZY TIME SERIES FORECASTING METHOD BASED ON GUSTAFSON-KESSEL FUZZY CLUSTERING[*]

Faruk ALPASLAN[a], Ozge CAGCAG[b]

Abstract

Fuzzy time series forecasting methods do not require constraints found in conventional approaches. In addition, due to uncertainty that they contain, many time series to be forecasted should be considered as fuzzy time series. Fuzzy time series forecasting models consist of three steps as fuzzification, identification of fuzzy relations and defuzzification. Although most of the time series encountered in real life contain seasonal component, only few of these fuzzy time series approaches analyze seasonal fuzzy time series. Even though all these studies have various advantages, their biggest disadvantage is to take into consideration only the fuzzy set having the highest membership value rather than the membership value of observations belonging to each fuzzy set. This situation conflicts to fuzzy set theory and causes the loss of information thus, negatively affects on the forecasting performance. In this study, a seasonal fuzzy time series forecasting model, in which Gustafson-Kessel fuzzy clustering technique in fuzzification stage is initially used and membership values are taken into account in both the determining fuzzy relations and the defuzzification stages is proposed. The proposed method is applied to real life seasonal time series and substantial results are obtained.

Keywords: Seasonal fuzzy time series, Gustafson-Kessel fuzzy clustering, membership value, forecasting.

JEL Classification: C53 - Forecasting and Prediction Methods.

Authors' Affiliation

[a] - Professor, University of Ondokuz Mayis, Turkey, e-mail: falpas@omu.edu.tr

[b] - Research assistant at the University of Ondokuz Mayis, Turkey, e-mail: ozge.cagcag@omu.edu.tr (corresponding author)

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1. Introduction

Many different approaches such as stochastic and non-stochastic models have been proposed in literature for the analysis of time series. In recent years, the use of non-stochastic models has become widespread. Fuzzy time series forecasting models do not require assumptions that stochastic models do. On the other hand, most of the time series encountered in real life should be considered as fuzzy time series due to the uncertainty that they contain and they should be analyzed with models appropriate to fuzzy set theory.

The conception of fuzzy time series was firstly put forward by Song and Chissom (1993a). Fuzzy time series methods consist of three steps. These are fuzzification, identification of fuzzified relations and defuzzification, respectively. Many studies on these three steps have been done in literature because of these steps have positive and negative impact on the forecasting performance of the method. While some of the approaches proposed in the literature involve first-order forecasting models, some of them involve high-order forecasting models. Song and Chissom (1993a, b 1994), Chen (1996), Yolcu et al. (2009) can be given as examples of first-order fuzzy time series forecasting models. Also, Chen (2002) and Aladag, et al. (2009) studies involve high-order fuzzy time series forecasting models. Although these models were effectively used in forecasting lots of fuzzy time series, fail to forecast fuzzy time series which contain seasonal component which can be frequently encountered in real life.

In this respect, Song (1999) proposed a method for the analysis of seasonal fuzzy time series. The model proposed by Song (1999) includes only lagged variable belonging to period. However, numerous time series include more complex relations apart from this structure. In an effort to forecast these types of time series, Egrioglu et al. (2009) proposed a partial high order fuzzy time series forecasting model based on SARIMA model. Although Egrioglu et al.'s approach has many advantages; it uses universal set partition in fuzzification step. The effect of interval lengths which were determined subjectively in fuzzification step on forecasting performance was presented in many studies in literature. In order to eliminate this problem, Uslu et al. (2010) developed the model proposed in and used fuzzy C-means method (FCM) which does not require universal set partition in fuzzification step.

Even though all these studies have various advantages, their biggest disadvantage is to take into consideration only the fuzzy set having the highest membership value rather than the membership value of observations belonging to each fuzzy set. This is contrary to fuzzy set theory and causes information loss thus affecting forecasting performance negatively. In literature, although there is only two methods which consider memberships values in determining fuzzy relations (Yu et al. 2010; Yolcu et al. 2011), these studies include first order fuzzy time forecasting method but is not used for forecasting seasonal fuzzy time series. In these approaches which consider membership values, if the high order models are used, the possible problem is the excessive input number of artificial neural network which is used in determining the relation.

In this study, a seasonal fuzzy time series forecasting model, in which Gustafson-Kessel fuzzy clustering technique in fuzzification stage is initially used and membership values are taken into account in both the determining fuzzy relations and the defuzzification stages is proposed. The proposed method is applied to real life seasonal time series.

The rest of the paper is organized as follows: Section 2 briefly describes SARIMA models which were used in determining the method order, the Gustafson-Kessel fuzzy clustering technique and ANNs techniques. In Section 3, basic definitions and notions of fuzzy time series are presented in summary. In Section 4, proposed method is explained in detail. The experimental results are presented in Section 5. Finally, in the last section, obtained results are discussed.

2. Review

2.1.SARIMA

When a time series with μ mean, than the model is expressed in equation (1)

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D(Z_t-\mu)=\theta(B)\Theta(B^s)a_t \quad (1)$$

Model parameters can be given as follows:

$$\phi(B)=(1-\phi_1 B-\cdots-\phi_p B^p) \quad (2)$$

$$\theta(B)=(1+\theta_1 B+\cdots+\theta_q B^q) \quad (3)$$

$$\Phi(B)=(1-\Phi_1 B^s-\cdots-\Phi_P B^{sP}) \quad (4)$$

$$\Theta(B)=(1+\Theta_1 B+\cdots+\Theta_Q B^{sQ}) \quad (5)$$

Detailed information on the model which is called seasonal autoregressive integrated moving average (SARIMA) and which is expressed as $\text{SARIMA}(p,d,q)(P,D,Q)_s$ can be obtained from Box and Jenkins (1976).

2.2.The Gustafson-Kessel fuzzy clustering technique

The algorithm of Gustafson-Kessel fuzzy clustering is firstly proposed in Gustafson and Kessel (1979). Let Σ_i be the covariance matrix of the cluster, c_i be the center of the ith cluster, u_{ij} be the membership degree and β be fuzziness index. For the *i*th cluster, its associated Mahalanobis distance is defined as

$$d^2(x_j, c_i, \Sigma_i) = (x_j - c_i)^T \Sigma_i^{-1} (x_j - c_i) \qquad (6)$$

The covariance matrices are computed as follow:

$$\Sigma_i = \frac{\Sigma_i^*}{\sqrt[p]{det(\Sigma_i^*)}} \qquad (7)$$

$$\Sigma_i^* = \frac{\sum_{j=1}^{n} u_{ij}(x_j - c_i)(x_j - c_i)^T}{\sum_{j=1}^{n} u_{ij}} \qquad (8)$$

The objective function is defined as

$$J(X, C, \Sigma, U) = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^{\beta} d^2(x_j, c_i, \Sigma_i) \qquad (9)$$

The objective function $J(X, C, \Sigma, U)$ is, then, minimized under the following constraints:

$$0 \leq u_{ij} \leq 1, \quad \forall i, j \qquad (10)$$

$$0 < \sum_{j=1}^{n} u_{ij} \leq n, \ \forall i \qquad (11)$$

$$\sum_{i=1}^{c} u_{ij} = 1 \qquad (12)$$

In this minimization problem, the center c_i and the membership degrees u_{ij} are updated according to the expressions given below.

$$c_i = \frac{\sum_{j=1}^{n} u_{ij}^{\beta} x_j}{\sum_{j=1}^{n} u_{ij}^{\beta}} \qquad (13)$$

$$u_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d(x_j, c_i)}{d(x_j, c_k)} \right)^{2/(\beta-1)}} \qquad (14)$$

2.3.Feed Forward Neural Network

Artificial neural networks (ANN) can be defined as the mathematical algorithm that is inspired by the biological neural networks (Gunay, Egrioglu and Aladag, 2007). Artificial neural networks are much more different than biological ones in terms of their structure and ability (Zurada, 1992). Artificial neural networks compose of a mathematical model (Zhang, Patuwo and Hu, 1998). The learning capability of an artificial neuron is achieved by adjusting the weights in accordance to the chosen learning algorithm. The basic architecture consists of three types of neuron layers: input, hidden, and output layers. In feed-forward networks, the signal flow is from input to output units, strictly in a feed-forward direction. Artificial neural network architectures are characterized by the following attributes:

Number of Layers: The artificial neurons are arranged in an input layer, one or more hidden layers, and an output layer.

Number of Neurons: The artificial neural network has to learn the features of the series for the analysis and forecasting of a fuzzy time series. As the number of neurons in the input and output layers are determined by the training patterns, the number of neurons in the hidden layers can then be chosen arbitrarily (see Figure 1). More artificial neurons imply more weighting matrices. Thus, from classical fields of application of artificial neural networks (e.g., pattern recognition), the well-known problem of over fitting must be considered.

Activation Function: The proper selection of activation function that enables curvilinear matching between input and output units, significantly affect the performance of the network.

Method of Training: The learning situations in neural networks may be classified into three distinct sorts. These are supervised learning, unsupervised learning, and reinforcement learning. In supervised learning, an input vector is presented at the inputs together with a set of desired responses, one for each node, at the output layer. The most widely used one is Back Propagation algorithm which updates weights based on the difference between available data and the output of the network. Learning parameter which is used in back propagation algorithm and which can be taken fixedly or updated in the algorithm dynamically, plays an important role in reaching optimal results.

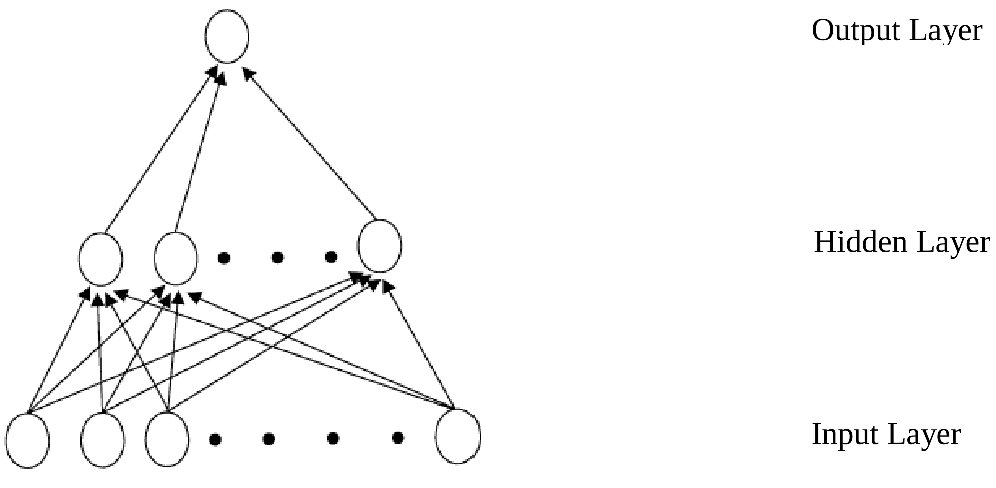

Figure 1. Architecture of multilayer feed forward neural network.

3. Fuzzy Time Series

The definition of fuzzy time series was firstly introduced by Song and Chissom (1993a). Basic definitions of fuzzy time series not including constraints such as linear model and observation number can be given as follows.

Definition 1 Fuzzy time series.

Let $Y(t)(t = \cdots, 0,1,2,\cdots)$, a subset of real numbers, be the universe of discourse by which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_1(t), \cdots$, then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

Definition 2 First order seasonal fuzzy time series forecasting model.

Let $F(t)$ be a fuzzy time series. Assume there exists seasonality in $\{F(t)\}$, first order seasonal fuzzy time series forecasting model:

$$F(t-m) \rightarrow F(t) \quad (15)$$

where *m* denotes the period.

Definition 3 High order fuzzy time series forecasting model.

Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $(t-1), F(t-2), \cdots, F(t-n)$, then this fuzzy logical relationship is represented by

$$F(t-n), \cdots, F(t-2), F(t-1) \rightarrow F(t) \quad (16)$$

and it is called the nth order fuzzy time series forecasting model.

Definition 4 First order bivariate fuzzy time series forecasting model.

Let F and G be two fuzzy time series. Suppose that $(t-1) = A_i$, $G(t-1) = A_k$ and $F(t) = A_j$. A bivariate fuzzy logical relationship is defined as $A_i, B_k \rightarrow A_j$, where A_i, B_k are referred to as the left hand side and A_j as the right hand side of the bivariate fuzzy logical relationship. Therefore, first order bivariate fuzzy time series forecasting model is as follows:

$$F(t-1), G(t-1) \rightarrow F(t) \quad (17)$$

Definition 5 High order partial bivariate fuzzy time series forecasting model.

Let F and G be two fuzzy time series. If $F(t)$ is caused by $F(t-m_1), \cdots, F(t-m_{k-1})$, $F(t-m_k), G(t-n_1), \cdots,$ $G(t-n_{l-1}), G(t-n_l)$, where m_i $(i = 1,2,\cdots,k)$ and n_j $(j = 1,2,\cdots,l)$ are integers $1 \leq m_1 < \cdots < m_k$, $1 \leq n_1 < \cdots < n_l$ then this FLR is represented by

$$\left.\begin{matrix} F(t-m_1), \cdots, F(t-m_{k-1}), F(t-m_k), \\ G(t-n_1), \cdots, G(t-n_{l-1}), G(t-n_l) \end{matrix}\right\} \rightarrow F(t) \quad (18)$$

4. Proposed Method

Although, there are numerous fuzzy time series approaches in literature, a few of these approaches intend to analyze seasonal fuzzy time series. Moreover all approaches which analyzed seasonal fuzzy time series used set number representing the only fuzzy set having the highest membership value of observations. This situation negatively affects the forecasting performance of method. In this study, we proposed a new seasonal fuzzy time series forecasting model. In our model, SARIMA is used in determination of the model, Gustafson-Kessel fuzzy clustering technique is used in fuzzification and ANN is used in determining fuzzy relations. Also in the proposed model, membership values are taken into account in both the determining fuzzy relations and the defuzzification stages.

The algorithm of the proposed method in this study is given below:

Algorithm

Step 1 The model order is defined by SARIMA

The time series concerned is analyzed by Box-Jenkins method after the model order is defined. Then we obtain residuals series (a_t). As an illustration let us suppose we have defined the model as SARIMA $(1,1,0)(0,1,1)_{12}$ via Box-Jenkins method. This implies that X_t will be a linear combination of the corresponding lagged variables. That is,

$$X_t = f(X_{t-1}, X_{t-2}, X_{t-12}, X_{t-13}, X_{t-14}, a_{t-12}) \quad (19)$$

Therefore, (k, l) representing the order of the model and the parameters $m_1, \cdots, m_k$ and $n_1, \cdots, n_l$ are determined based on the inputs of the SARIMA model. Accordingly k and l are defined as 5 and 1 respectively. Then the model will be $(5,1)^{th}$-order partial bivariate fuzzy time series forecasting model and the fuzzy relationship can be given as follows;

$$\left.\begin{matrix} F(t-1), F(t-2), F(t-12), \\ F(t-13), F(t-14), G(t-12) \end{matrix}\right\} \rightarrow F(t) \quad (20)$$

This implies $m_1 = 1, m_2 = 2, m_3 = 12, m_4 = 13, m_5 = 14, n_1 = 12$, $F(t)$ denotes the fuzzified time series X_t and $G(t)$ denotes the fuzzified residual series a_t.

Step 2 Data set of lagged variables is created.

Depending on the model order defined in previous step, for each time series which should be included in the model X_t, and residual series a_t for each lagged variables are lagged less than order of lagged variables and data set is created. In other words, when a model given in equation (20) is considered, lagged variables data set will include $X_t, X_{t-1}, X_{t-11}, X_{t-12}, X_{t-13}, a_{t-11}$.

Step 3 Data set of lagged variables is clustered via Gustafson-Kessel fuzzy clustering.

The number of fuzzy set is determined with c where $2 \leq c \leq n$ and n is the number of observation. Data set which covers the delays in times series is clustered via Gustafson-

Kessel fuzzy clustering method. Thus, fuzzy set centers for each lagged variables constituting data set and membership values showing order of observations belonging to fuzzy sets for each observation are obtained. In this step, fuzzy sets are sorted according to set centers represented with $v_r,\ r = 1,2,\cdots,c$ and $L_r, r = 1,2,\cdots,c$ fuzzy sets are obtained.

Step 4 Fuzzy relations are determined via Feed Forward Artificial Neural Networks (ANN).

The number of neurons in input and output layer of feed forward artificial neural network used in determining fuzzy relations equals to number of fuzzy set (c). The number of neurons in hidden layer is determined by trial and error. Here, the point to take into consideration is that hidden layer unit number should be selected in a way that not losing generalization ability of feed forward artificial neural network. The architecture of feed forward artificial neural network having two hidden layers for a model including seven sets is presented in Figure 3. In Figure 2, $\mu_{L_i}(DS(t))$ represents the membership value of lagged data set belonging to i^{th} fuzzy set at t time. Moreover, while membership value of observation of lagged data set belonging to c number fuzzy set at t time constitutes the inputs of ANN; membership value of observation of lagged data set belonging to c number fuzzy set at t time constitutes the outputs of ANN.

In all layers of feed forward artificial neural networks which is used in determining fuzzy relation and whose architectural structure is exemplified above, logistic activation function given in (21) equation is used.

$$f(x) = (1 + \exp(-x))^{-1} \qquad (21)$$

Feed forward artificial neural networks are trained according to Levenberg-Marquardt learning algorithm and optimal weights are obtained. Trained artificial neural network learned the relation between consecutive time series observations and membership values of sets.

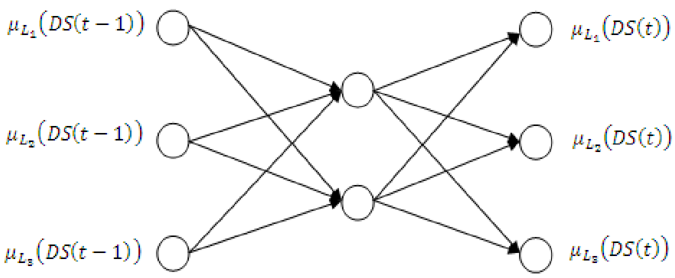

Figure 2. Architecture of feed forward artificial neural network for three sets

Step 5 Defuzzification of forecasts.

In order to obtain fuzzy forecasts of fuzzy time series at t time, membership values of observations belonging to fuzzy sets at $t-1$ time depending on $v_r,\ r=1,2,\cdots,c$ fuzzy set center which was obtained from Gustafson-Kessel fuzzy clustering method were determined and then these membership values were entered to feed forward artificial neural networks as inputs and thus outputs of feed forward artificial neural networks are created. These outputs represent the membership values for fuzzy forecast of observation at t time. It must be noted that the sum of membership values obtained for fuzzy forecast value is not equal to 1, contrary to Gustafson-Kessel fuzzy clustering method. In defuzzification step, membership values of fuzzy forecasts are converted to weights as in (22) and defuzzified forecast is obtained as in (23).

$$w_{it}=\frac{\hat{u}_{it}}{\hat{u}_{1t},+\hat{u}_{2t}+\cdots\hat{u}_{ct}} \qquad (22)$$

$$\hat{X}_t=\sum_{i=1}^{c} w_{it}v_i \qquad (23)$$

Here, $\hat{u}_{it}$ are the membership values of observation obtained from outputs of feed forward artificial neural network at t time, and w_{it} are the weights used in determining fuzzified forecasts.

5. Application

The proposed method was applied to time series of “the amount of sulfur dioxide in Ankara province between March 1994 and April 2006 (ANSO)”. The graph of ANSO time series is presented in Figure 4.

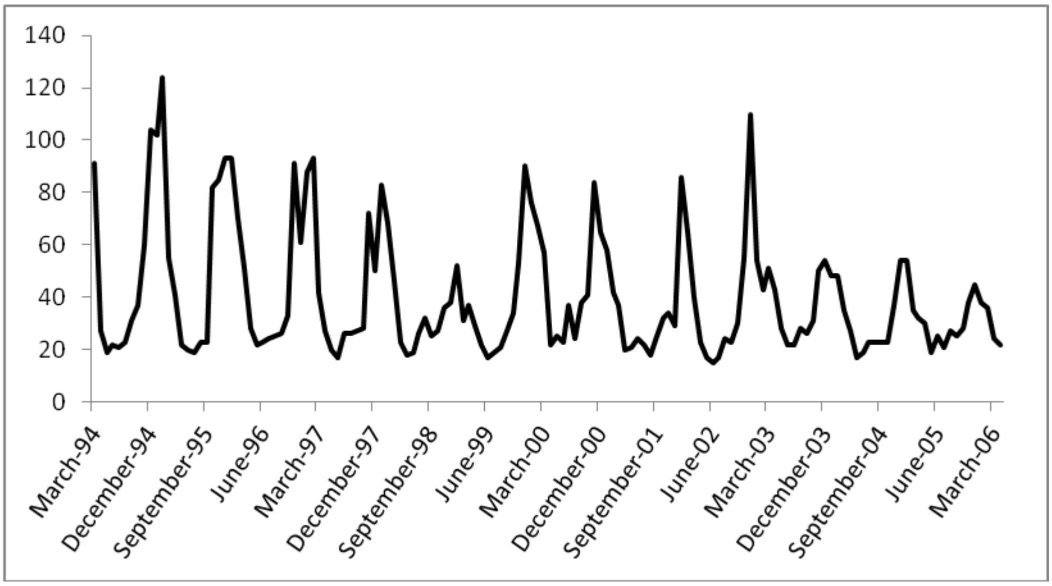

Figure 4. The time series data of the amount of SO_2 in Ankara.

In order to evaluate the performance of the proposed method, the last 10 observations were taken as test set and obtained results were compared with some conventional and alternative time series methods. In the application, in order to determine the order of fuzzy time series forecasting model, crisp time series is analyzed using Box-Jenkins method and optimal SARIMA model is determined and residual time series (a_t) as well as X_t time series are obtained. In this step, optimal model for ANSO time series was $SARIMA(1,1,0)(0,1,1)_{12}$. As a linear function of X_t, this model can be expressed as;

$$X_t = f(X_{t-1}, X_{t-2}, X_{t-12}, X_{t-13}, X_{t-14}, a_{t-12}) \quad (24)$$

Thus, the model will be (5, 1)th order partial high order fuzzy time series forecasting model where $k = 5$ and $l = 1$. This model can be expressed as;

$$\left.\begin{matrix} F(t-1), F(t-2), F(t-12), \\ F(t-13), F(t-14), G(t-12) \end{matrix}\right\} \rightarrow F(t) \quad (25)$$

After determining the model order of partial high order model, lagged variables data set for each lagged variable that should be included in the model is created. Lagged variables data set for $(5,1)^{th}$ order partial model is created using $X_t, X_{t-1}, X_{t-11}, X_{t-12}, X_{t-13}, a_{t-11}$ lagged variables. Here, it must be noted that lagged variables data set consists of one step leaded variable in partial high order fuzzy time series forecasting model given in (20). Created data set is clustered via Gustafson-Kessel fuzzy clustering. Clustering is applied to all lagged variable data sets together. In this step, data set is clustered by shifting the number of sets 5 to 15. Membership values of observations belonging to each fuzzy set are also determined via Gustafson-Kessel fuzzy clustering method. The relationship between these membership values, in other words, the number of neurons in hidden layer of feed forward artificial neuron network which is used in determining fuzzy relation were shifted between 1 and 15. In the light of this information, $11 \times 15 = 165$ different analyses were done and Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE) were used as performance evaluation criteria.

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(x_t - \hat{x}_t)^2} \quad (26)$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{x_t - \hat{x}_t}{x_t}\right| \quad (27)$$

where x_t, and $\hat{x}_t$, T represent crisp time series, defuzzified forecasts, and the number of forecasts, respectively. The algorithm of the proposed method is coded in Matlab version 7.9.

In consequence of all analyses, the best forecasting performance was obtained in the case in which the number of set is 8, the hidden layer unit number is 10 in the determination of fuzzy relation stage. Results obtained from the proposed method and results of some other methods are summarized in Table 1. Also Figure 1 presents the graph of the results obtained from the proposed method and real time series for test data.

When the Table 1 is examined it is seen that proposed method has the better forecasting performance than some other conventional and seasonal fuzzy time series approaches.

Table 1. Result of methods

Data Set	SARIMA	WMES	Song (1999)	Egrioglu et al. (2009)	Uslu et al. (2010)	Proposed Method
21	22.93	15.40	41.6667	20.00	22.7536	25.8477
27	22.35	16.11	27.5000	30.00	22.7536	25.1923
25	23.61	17.77	41.6667	20.00	22.7536	27.0025
28	28.81	25.12	41.6667	30.00	22.7536	25.8477
38	46.97	41.11	41.6667	30.00	42.0558	37.0206
45	54.62	46.12	46.7857	50.00	42.0558	38.0066
38	58.13	49.80	45.0000	40.00	42.0558	39.0811
36	46.99	44.24	46.7857	30.00	42.0558	38.0066
24	37.85	31.96	46.7857	30.00	22.7336	25.1583
22	24.76	18.39	27.5000	20.00	22.7536	22.8639
RMSE	9.62	7.11	12.74	4.56	3.66	3.04
MAPE	0.23	0.22	0.40	0.13	0.11	0.08

WMES: Winters' Multicaptive Exponential Smoothing

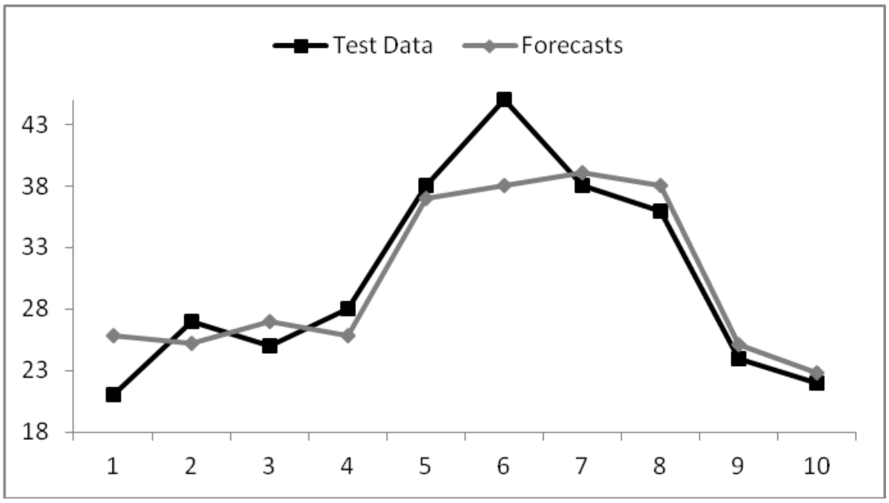

Figure 5. The graph of the results obtained from the proposed method and real time series.

6. Discussion and Conclusion

Various approaches have been proposed for forecasting of fuzzy time series. Although numerous first and high order fuzzy time series forecasting models have been put forward in literature, these models are insufficient in the analysis of seasonal fuzzy time series. The approaches proposed in literature have some advantages but these approaches have some significant disadvantages. One of the most significant disadvantages of these models is that they ignore membership values in analysis process. In this study, to overcome these kind of problems, a seasonal fuzzy time series forecasting model, in which Gustafson-Kessel fuzzy clustering technique in fuzzification stage is initially used and membership values are taken into account in both the determining fuzzy relations and the defuzzification stages is proposed. The proposed method in this study has some advantages and exhibits superior forecasting performance.

In future studies, different clustering techniques can be implemented in fuzzification step and different types ANN structures that may provide more effective gains in the determination of fuzzy relations can be attempted.

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